**Homework 3. Solutions**

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**Ex. 1.** As shown in the figure, a particle is moving anti-clockwise in a uniform circular motion. The mass of the particle is . The velocity of the particle at point A and point B are . **Please describe:** the impulse of the net external force exerted on the particle during the time interval from point A to point B in respect to unit vectors (the unit vectors in the -direction and -direction).



Solution:

At point A, the momentum is:

The change of momentum is:

Using the impulse-momentum theorem, the impulse of the net force from A to B is:

Ex. 2.

Suppose that there is a basketball fall freely (friction with air is ignored) from a height of . The mass of the particle is , it rebounds (elastic rebound) vertically after it impacts the ground. The duration of the collision is 0.019 s. (a) Describe the kinetic energy and the potential energy of the ball just after the ball is released from height h. (b) Describe the kinetic energy and the potential energy of the ball when impact the ground. (d) describe the momentum of the ball (magnitude in respect with m, g, h and direction ) just before the collision and just after the collision . (d) Calculate the magnitude of the average force exerted by the basketball on the ground during the collision and give its direction.

**Elastic rebound:** The magnitude of the velocity vector of the ball is the same just before and just after the impact.

**Solution:**

(a)We consider the potential energy of the ball is zero at the ground (this choice is arbitrary). Just after the ball is released, its potential energy is , its kinetic energy is (because its velocity is zero).

(b)After falling a height of h, the potential energy of the ball is , its kinetic energy is . The only force exerted on the ball during its falling is the weight, which is a conservative force, which means

(c) The momentum of the ball just before the impact is directed downward, its magnitude is: . , thus . The momentum of the ball just after the impact is upward and its magnitude is .

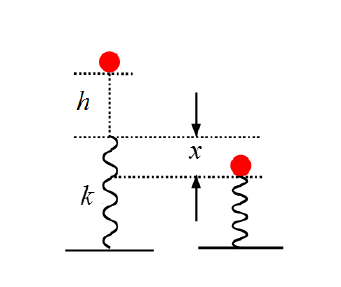
(d)The average force exerted by the ground on the ball during the impact is , it is directed upward and its magnitude is . Using the Newton’s third law, the average force exerted by the ball on the ground has magnitude and is directed downward. Using , we obtain:

About checking the units, (easy to remember from the Newton’s second law)

**Ex. 3** As shown in the figure, a particle with mass falls freely from a height of above the spring (spring coefficient is ). **Please find:** the maximum length the spring will be compressed. Friction is ignored (the only forces considered are conservative).

For symbols, please to use: is the gravitational potential energy of the ball just after falling, is the kinetic energy of the ball just after falling, is the gravitational potential energy of the ball at the maximum length the spring is compressed, the elastic potential energy ball at the maximum length the spring is compressed,the kinetic energy of the ball at the maximum length the spring is compressed

**Hint:** At the maximum length the spring is compressed, the velocity of the ball will be zero.



**Solution:**

The mechanical energy of the ball is constant (friction is ignored, the only forces considered are conservative):

With and , we obtain:

We choose for gravitational potential energy equals zero the height of the ball at the maximum length the spring is compressed (we could choose any height), thus:

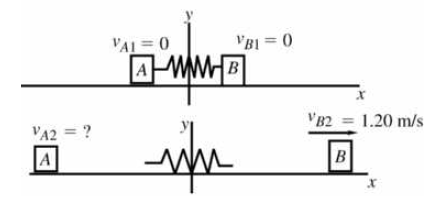
At the maximum length of spring compression, the elastic potential energy is:

because is also the distance from equilibrium position of the spring. We obtain a second degree equation to solve.

which the solution (the second solution is not considered) is:

**Ex. 4.**

Block A in the figure has mass 1.00 kg, and block B has mass 3.00 kg. The blocks are forced together, compressing a spring S between them; then the system is released from rest on a level, frictionless surface. The spring, which has negligible mass, is not fastened to either block and drops to the surface after it has expanded. Block B acquires a speed of (a) What is the x-component of the velocity vector of block A? (b) How much potential energy was stored in the compressed spring?

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**Solution:**

Using the conservation of the momentum, we obtain:

where is the mass of block A, is the mass of block A, and are the velocity vector of block B before and after the release, and are the velocity vector of block B before and after the release.

Using , we obtain:

(b) Only the spring force (a conservative force) does work (during an horizontal motion, the gravitational force doesn’t do work), the table is frictionless. Conservation of the mechanical energy, i.e.

When the blocks are at rest, the total kinetic energy of the system is . The potential energy stored in the spring is

When the blocks are in motion, the total kinetic energy of the system is:

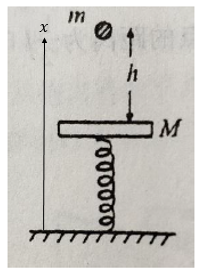
The potential energy stored in the spring is: . We obtain:

**Ex. 5. An elastic collision.**

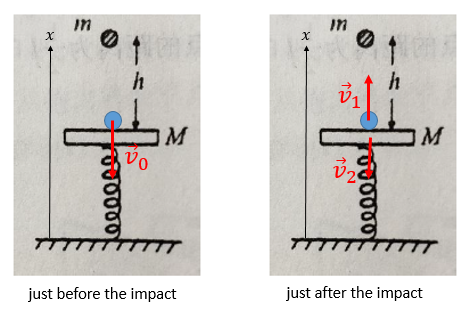
A spring is fixed vertically on the ground. The stiffness (spring coefficient) of the spring is .A steel plate with a mass of is tied on the spring. Suppose a ball with a mass of () falls free from a height of h above the plate M and collides with the plate elastically. Please find: (1) describe the velocity at which the ball rebound from the steel plate and the velocity of the plate just after the impact (direction and magnitude in respect with ) (2) what is the height the ball can reach after the collision. (3) What is the length is the spring compressed during this procedure?

Please to use the following symbols: and are the velocity vector and the momentum of the ball just before the impact with the steel plate, : and are the velocity and the momentum of the ball just after the impact with the steel plate, and are the velocity and the momentum of the steel plate after the impact.

**Hint:** considering the system “ball + plate”, the external force during the collision is zero (the momentum of the system is conserved), and the collision is elastic (the kinetic energy of the system is conserved).



**Solution:**

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(1)

Conservation of momentum (the momentum of the steel plate before the impact is zero):

Using the x-components of the velocity vectors, we obtain:

Conservation of the kinetic energy of the system “ball+plate” (kinetic energy of the plate before the impact is zero):

Which can be in the form:

The both equations of conservation permit to find and . First, the equation of conservation of kinetic energy is arranged as follows:

The equation of the equation of conservation of momentum is arranged as follows:

We divide these two arranged equations and obtain:

Thus, we substitute in the conservation of the momentum equation, we obtain:

Finally we find the expression of

About the direction of , here, the +x-direction is upward, and is directed downward so . , which means , thus is directed upward.

Using (same explanation that for in exercise 2) and , we obtain:

The velocity is simply obtained using,

because .

Using and , we obtain:

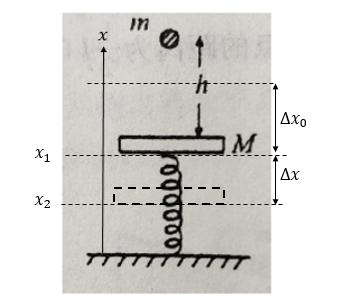
is directed downward, is directed upward, is directed downward.

Take care, that x-component of the velocity vectors can be positive or negative (its depends to the direction of the vectors), but their magnitude is always positive: , , .

(2) Height of the ball after the collision. The potential energy corresponding to this height is:

Where the potential energy equals zero is chosen to be where there is collision between the plate and the ball. The friction is ignored, only the gravitational force does work on the ball after the collision, the mechanical energy of the ball is conserved. At the maximum height, its velocity is zero, so its kinetic energy is zero. We find :

(3) Let says is the x-position of the plate just after the impact and the x-position of the plate at the maximum length of compression, thus the additional length of compression of the spring is:



At the maximum length the spring is compressed from equilibrium, the plate has velocity equals zero, so it has no kinetic energy. The only forces which does work on the plate (spring force and gravitational force) are conservative force. Its mechanical energy is conserved. When the system plate + spring is at equilibrium, the spring is compressed of a length from its equilibrium position without the plate (i.e. the spring is not stretched nor compressed). Using the Newton’s first law on the plate (the forces exerted are its weight and the spring force), we obtain:

After the impact by the ball, the spring is compressed to another length (so its total compression , length is ). The change of potential energy associated with the spring force exerted on the plate when the spring is compressed of a distance to a distance is:

At the compression length the plate has no velocity, so it has no kinetic energy. Just after the impact, the kinetic energy of the plate is:

The change of gravitational potential energy of the plate is, from to :

The mechanical energy is conserved (only conservative forces are involved, the friction is ignored), thus:

We obtain:

(you don’t need to give so much details in examination, I just give you them to be sure it is understood)